

6

CHAPTER SUMMARY

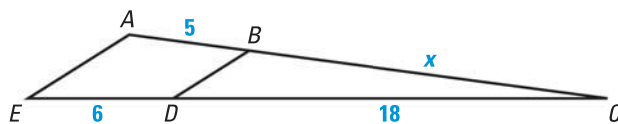
BIG IDEAS

For Your Notebook

Big Idea 1

Using Ratios and Proportions to Solve Geometry Problems

You can use properties of proportions to solve a variety of algebraic and geometric problems.



For example, in the diagram above, suppose you know that $\frac{AB}{BC} = \frac{ED}{DC}$. Then you can write any of the following relationships.

$$\frac{5}{x} = \frac{6}{18}$$

$$5 \cdot 18 = 6x$$

$$\frac{x}{5} = \frac{18}{6}$$

$$\frac{5}{6} = \frac{x}{18}$$

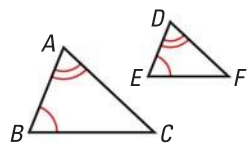
$$\frac{5+x}{x} = \frac{6+18}{18}$$

Big Idea 2

Showing that Triangles are Similar

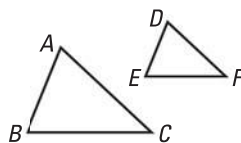
You learned three ways to prove two triangles are similar.

AA Similarity Postulate



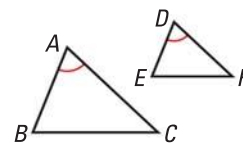
If $\angle A \cong \angle D$ and $\angle B \cong \angle E$, then $\triangle ABC \sim \triangle DEF$.

SSS Similarity Theorem



If $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$, then $\triangle ABC \sim \triangle DEF$.

SAS Similarity Theorem



If $\angle A \cong \angle D$ and $\frac{AB}{DE} = \frac{AC}{DF}$, then $\triangle ABC \sim \triangle DEF$.

Big Idea 3

Using Indirect Measurement and Similarity

You can use triangle similarity theorems to apply indirect measurement in order to find lengths that would be inconvenient or impossible to measure directly.

Consider the diagram shown. Because the two triangles formed by the person and the tree are similar by the AA Similarity Postulate, you can write the following proportion to find the height of the tree.

$$\frac{\text{height of person}}{\text{length of person's shadow}} = \frac{\text{height of tree}}{\text{length of tree's shadow}}$$

You also learned about dilations, a type of similarity transformation. In a dilation, a figure is either enlarged or reduced in size.



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CHAPTER REVIEW

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- Multi-Language Glossary
- Vocabulary practice

REVIEW KEY VOCABULARY

For a list of postulates and theorems, see pp. 926–931.

- ratio, p. 356
- proportion, p. 358
means, extremes
- geometric mean, p. 359
- scale drawing, p. 365
- scale, p. 365
- similar polygons, p. 372
- scale factor of two similar polygons, p. 373
- dilation, p. 409
- center of dilation, p. 409
- scale factor of a dilation, p. 409
- reduction, p. 409
- enlargement, p. 409

VOCABULARY EXERCISES

Copy and complete the statement.

1. A $\underline{\quad ? \quad}$ is a transformation in which the original figure and its image are similar.
2. If $\triangle PQR \sim \triangle XYZ$, then $\frac{PQ}{XY} = \frac{?}{YZ} = \frac{?}{?}$.
3. **WRITING** Describe the relationship between a ratio and a proportion. Give an example of each.

REVIEW EXAMPLES AND EXERCISES

Use the review examples and exercises below to check your understanding of the concepts you have learned in each lesson of Chapter 6.

6.1 Ratios, Proportions, and the Geometric Mean

pp. 356–363

EXAMPLE

The measures of the angles in $\triangle ABC$ are in the extended ratio of 3 : 4 : 5. Find the measures of the angles.

Use the extended ratio of 3 : 4 : 5 to label the angle measures as $3x^\circ$, $4x^\circ$, and $5x^\circ$.

$$3x^\circ + 4x^\circ + 5x^\circ = 180^\circ \quad \text{Triangle Sum Theorem}$$

$$12x = 180 \quad \text{Combine like terms.}$$

$$x = 15 \quad \text{Divide each side by 12.}$$

So, the angle measures are $3(15^\circ) = 45^\circ$, $4(15^\circ) = 60^\circ$, and $5(15^\circ) = 75^\circ$.

EXERCISES

4. The length of a rectangle is 20 meters and the width is 15 meters. Find the ratio of the width to the length of the rectangle. Then simplify the ratio.
5. The measures of the angles in $\triangle UVW$ are in the extended ratio of 1 : 1 : 2. Find the measures of the angles.
6. Find the geometric mean of 8 and 12.

EXAMPLES 1, 3, and 6

on pp. 356–359
for Exs. 4–6

6.2 Use Proportions to Solve Geometry Problems

pp. 364–370

EXAMPLE

In the diagram, $\frac{BA}{DA} = \frac{BC}{EC}$. Find BD .

$$\frac{x+3}{3} = \frac{8+2}{2}$$

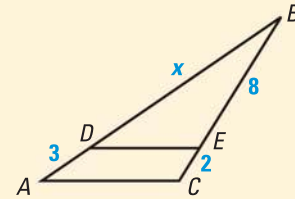
Substitution Property of Equality

$$2x + 6 = 30$$

Cross Products Property

$$x = 12$$

Solve for x .

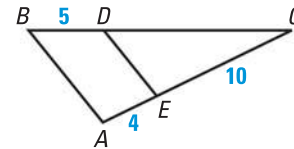
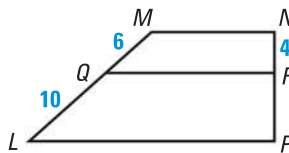


EXERCISES

Use the diagram and the given information to find the unknown length.

7. Given $\frac{RN}{RP} = \frac{QM}{QL}$, find RP .

8. Given $\frac{CD}{DB} = \frac{CE}{EA}$, find CD .



EXAMPLE 2

on p. 365
for Exs. 7–8

6.3 Use Similar Polygons

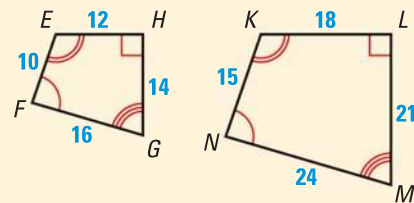
pp. 372–379

EXAMPLE

In the diagram, $EHGF \sim KLMN$. Find the scale factor.

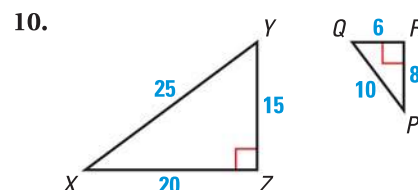
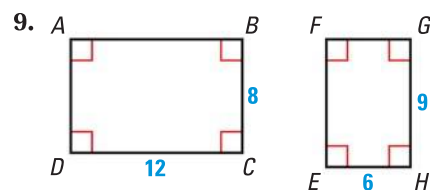
From the diagram, you can see that \overline{EH} and \overline{KL} correspond. So, the scale factor

of $EHGF$ to $KLMN$ is $\frac{EH}{KL} = \frac{12}{18} = \frac{2}{3}$.



EXERCISES

In Exercises 9 and 10, determine whether the polygons are similar. If they are, write a similarity statement and find the scale factor.



EXAMPLES 2 and 4

on pp. 373–374
for Exs. 9–11

11. **POSTERS** Two similar posters have a scale factor of 4:5. The large poster's perimeter is 85 inches. Find the small poster's perimeter.

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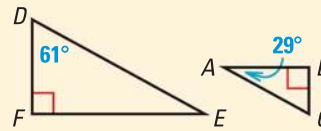
CHAPTER REVIEW

6.4 Prove Triangles Similar by AA

pp. 381–387

EXAMPLE

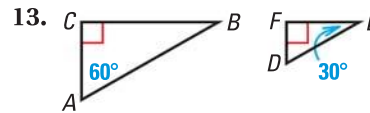
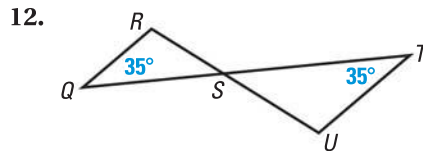
Determine whether the triangles are similar. If they are, write a similarity statement. Explain your reasoning.



Because they are right angles, $\angle F \cong \angle B$. By the Triangle Sum Theorem, $61^\circ + 90^\circ + m\angle E = 180^\circ$, so $m\angle E = 29^\circ$ and $\angle E \cong \angle A$. Then, two angles of $\triangle DFE$ are congruent to two angles of $\triangle CBA$. So, $\triangle DFE \sim \triangle CBA$.

EXERCISES

Use the AA Similarity Postulate to show that the triangles are similar.



14. **CELL TOWER** A cellular telephone tower casts a shadow that is 72 feet long, while a tree nearby that is 27 feet tall casts a shadow that is 6 feet long. How tall is the tower?

EXAMPLES 2 and 3

on pp. 382–383 for Exs. 12–14

6.5 Prove Triangles Similar by SSS and SAS

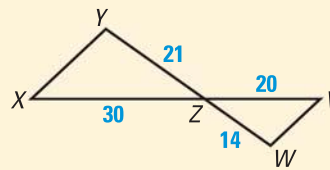
pp. 388–395

EXAMPLE

Show that the triangles are similar.

Notice that the lengths of two pairs of corresponding sides are proportional.

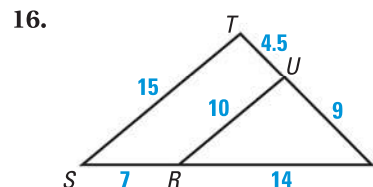
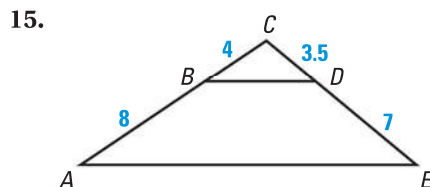
$$\frac{WZ}{YZ} = \frac{14}{21} = \frac{2}{3} \quad \frac{VZ}{XZ} = \frac{20}{30} = \frac{2}{3}$$



The included angles for these sides, $\angle XZY$ and $\angle VZW$, are vertical angles, so $\angle XZY \cong \angle VZW$. Then $\triangle XYZ \sim \triangle VWZ$ by the SAS Similarity Theorem.

EXERCISES

Use the SSS Similarity Theorem or SAS Similarity Theorem to show that the triangles are similar.



EXAMPLE 4

on p. 391 for Exs. 15–16

6.6 Use Proportionality Theorems

pp. 397–403

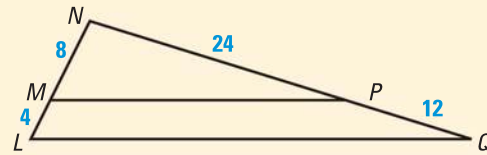
EXAMPLE

Determine whether $\overline{MP} \parallel \overline{LQ}$.

Begin by finding and simplifying ratios of lengths determined by \overline{MP} .

$$\frac{NM}{ML} = \frac{8}{4} = \frac{2}{1} \qquad \frac{NP}{PQ} = \frac{24}{12} = \frac{2}{1}$$

Because $\frac{NM}{ML} = \frac{NP}{PQ}$, \overline{MP} is parallel to \overline{LQ} by Theorem 6.5, the Triangle Proportionality Converse.



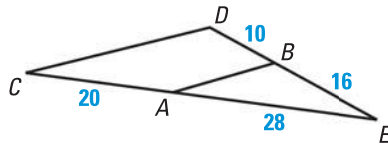
EXERCISES

Use the given information to determine whether $\overline{AB} \parallel \overline{CD}$.

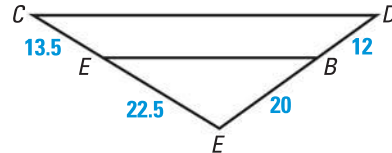
EXAMPLE 2

on p. 398
for Exs. 17–18

17.



18.



6.7 Perform Similarity Transformations

pp. 409–415

EXAMPLE

Draw a dilation of quadrilateral $FGHJ$ with vertices $F(1, 1)$, $G(2, 2)$, $H(4, 1)$, and $J(2, -1)$. Use a scale factor of 2.

First draw $FGHJ$. Find the dilation of each vertex by multiplying its coordinates by 2. Then draw the dilation.

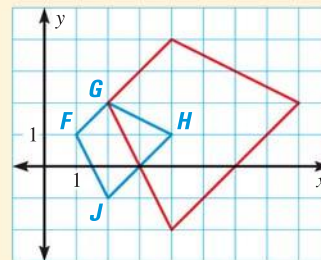
$$(x, y) \rightarrow (2x, 2y)$$

$$F(1, 1) \rightarrow (2, 2)$$

$$G(2, 2) \rightarrow (4, 4)$$

$$H(4, 1) \rightarrow (8, 2)$$

$$J(2, -1) \rightarrow (4, -2)$$



EXERCISES

Draw a dilation of the polygon with the given vertices using the given scale factor k .

19. $T(0, 8)$, $U(6, 0)$, $V(0, 0)$; $k = \frac{3}{2}$

20. $A(6, 0)$, $B(3, 9)$, $C(0, 0)$, $D(3, 1)$; $k = 4$

21. $P(8, 2)$, $Q(4, 0)$, $R(3, 1)$, $S(6, 4)$; $k = 0.5$

EXAMPLES 1 and 2

on pp. 409–410
for Exs. 19–21